Image Fusion: Beyond Wavelets

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Objectives

- The aim of this talk is threefold.
- First, I shall introduce the problem of **image fusion** and its role in modern signal processing.
- Next, I shall discuss **wavelets** from a mathematical point of view.
- Finally, I will show how wavelets offer a powerful technique in image fusion, and some recent work on these fusion algorithms.

Image Data

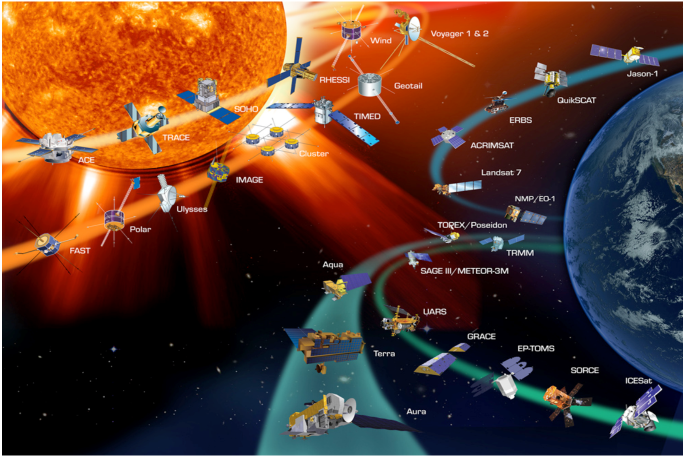
- It's a cliché: we live in an era of BIG DATA.
- Consider, for example, the variety of imaging techniques available for satellite imaging devices: RADAR, LIDAR, SONAR, visible, infared, gamma, multispectral, hyperspectral, panchromatic, etc.
- Each of these types of image data focuses on different features such as sharp edges, floral distribution, or mineral composition.

Central Problem of Image Fusion:

Combine these disparate images into one, which captures the best features of each individual component.

Why Image Fusion?

- NASA has hundreds of satellites in orbit:



- These take images in a variety of styles and resolutions. How to synthesize these?

Landsat 7 Satellite

- The Landsat 7 satellite orbits the earth, producing 8 bands of images. Bands 1-7 are multispectral. Band 8 is panchromatic. Let's look at some images taken in 2000, over Hasselt, Belgium.

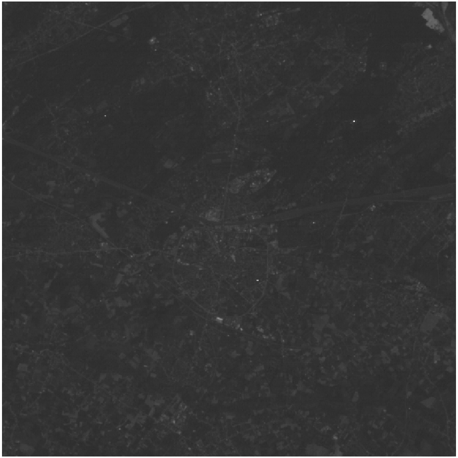


Figure: Band 1 of Landsat 7 (multispectral)

Landsat 7 Satellite

Figure: Band 8 of Landsat 7 (panchromatic)

- Harmonic analysis studies decompositions of functions into elementary pieces.
- The first and still canonical example of this approach is Fourier series:

Theorem

(Dirichlet) Suppose $f \in L^1[0, 2\pi]$ is differentiable at $x \in (0, 1)$.

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}, \text{ where } c_n = \frac{1}{2\pi} \int_0^{2\pi} f(y) e^{-iny} dy.$$

- So, we can decompose a “nice” function into a series that describes particular aspects of its behavior.
- Fourier series emphasize frequency content, so functions like sums of $\sin(x)$ and $\cos(x)$ are particularly well-represented in this system.

Wavelets

- There are other decompositions that emphasize other aspects of a function. Wavelets are an example of such a decomposition method.
- While Fourier series decomposes with respect to *frequency*, wavelets decompose with respect to *location and scale*:

Theorem

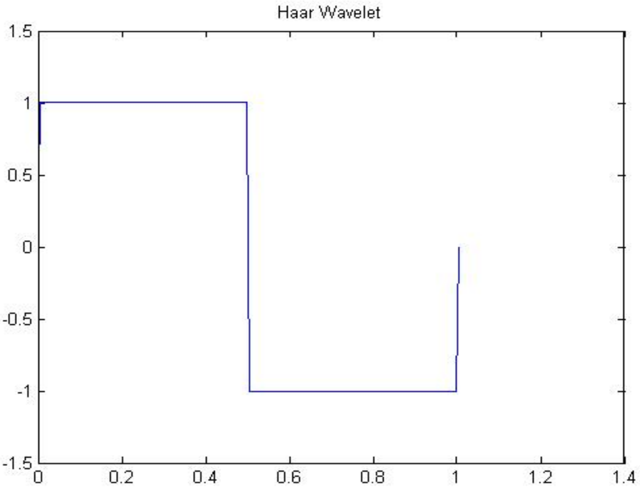
For a suitably chosen wavelet function ψ, we may decompose any $f \in L^2(\mathbb{R})$ as

$$f(x) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} c_{k,j} 2^{-\frac{j}{2}} \psi(2^{-j}x - k), \text{ where } c_{k,j} = 2^{-\frac{j}{2}} \int_{\mathbb{R}} f(y)\psi(2^{-j}y - k)dy$$

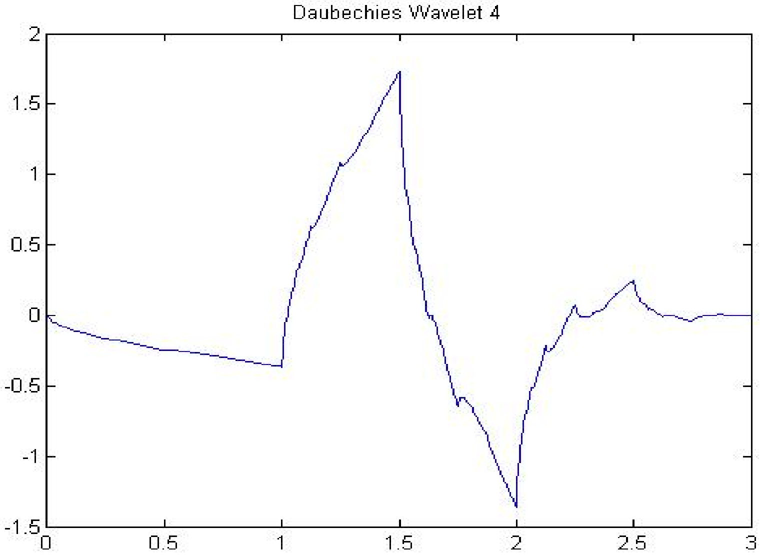
- Notice that our sum indexes over k, j. Changing k *translates* ψ. Changing j *dilates* ψ, picking up more local behavior ($j < 0$) or more global behavior ($j > 0$).

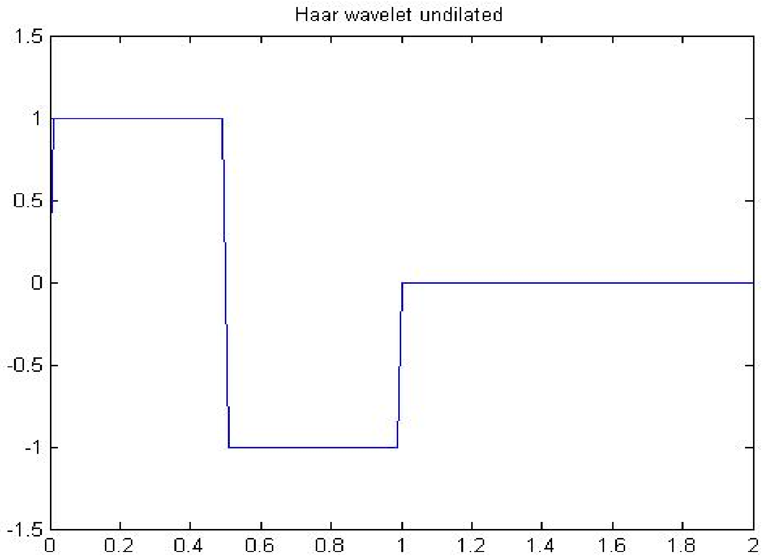
Choices for ψ

- Many choices of wavelet function ψ can be constructed mathematically, but a few are particularly well-used in applications.

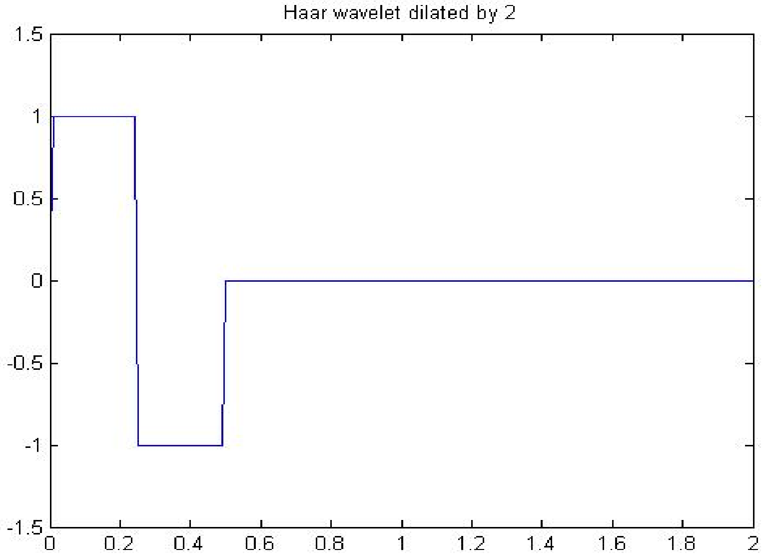


Choices for ψ

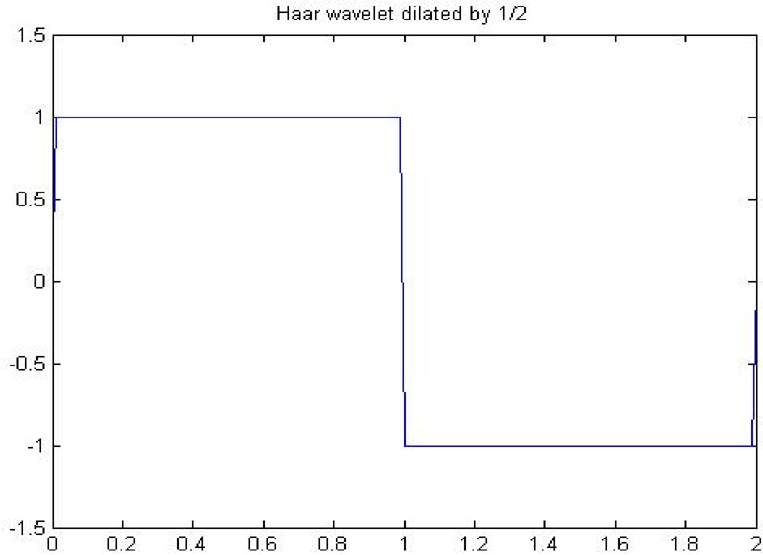




Plot of Haar wavelet $\psi(x)$.



Plot of $\psi(2x)$.



Plot of $\psi(\frac{x}{2})$.

Wavelets are good for Images

- As mentioned, functions of an oscillatory nature are well-represented by partial sums of their Fourier series.
- Functions representing images are usually well-represented by partial sums of wavelet decompositions.
- This is so much so that the standard image compression algorithm JPEG2000 is wavelet-based!
- The scale and translation information succinctly captures the essence of many images.

Wavelets+Fusion

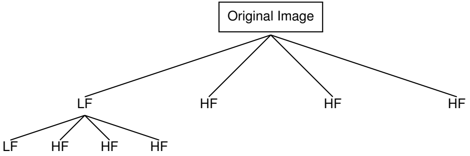
- Can we use wavelets for our problem in image fusion?

- First, we note that the wavelet decomposition can be implemented numerically to decompose an image.

- The *discrete wavelet transform* resolves an image according to
  1. "high frequency" features (building edges, rivers, sharp discontinuities).
  2. "low frequency" features (textures, variation in flora, soft transitions).

Using Algorithm

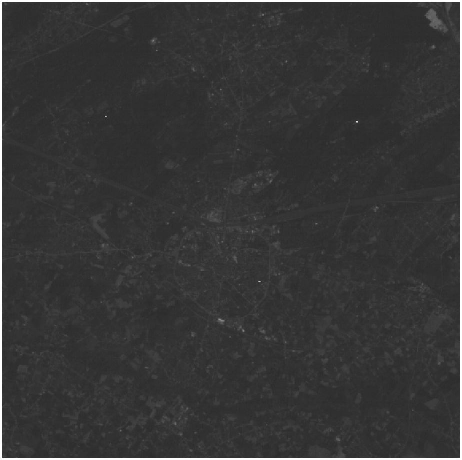
- This decomposition is *iterative*. In the case of two dimensions (appropriate for images), the initial signal is first decomposed into four coefficients.
- One of these coefficients represents pure low frequency features (LF), the other three hybrid high and low frequency features and pure high frequency features (HF). The LF coefficient is then further decomposed.
- This gives a nice tree structure, seen below for two levels of decomposition.



Fusion Algorithm

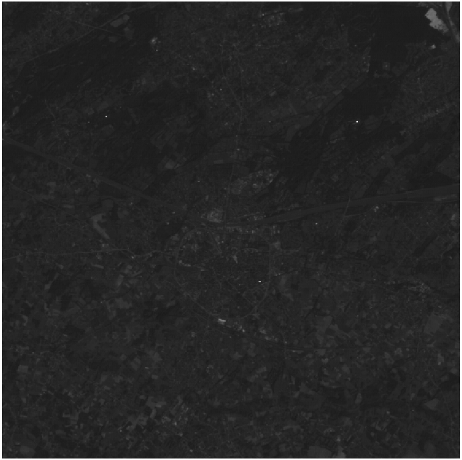
- We can exploit this knowledge of how wavelets decompose an image.
- Indeed, we shall perform our fusion in the wavelet domain by manipulating the wavelet coefficients of our images, then recovering the original image by applying an inverse transform.
- This lets us use the wavelet transform's separation of high frequency features (building edges, rivers, sharp discontinuities) and low frequency features (textures, variation in flora, soft transitions) to take the best features from each image and put them together in a new one.
- The development of these algorithms is joint work with Tim Doster and Wojtek Czaja.

Data (2000 DFC) - Hasselt, Belgium - Landsat 7



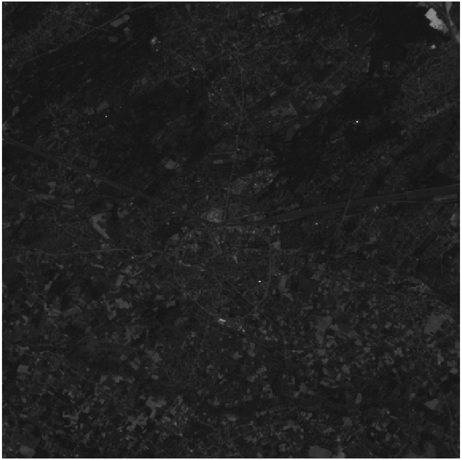
Band Number	1
Spectral Window (nm)	450-515
Spatial Resolution (m)	30
Entropy	3.9904

Data (2000 DFC) - Hasselt, Belgium - Landsat 7



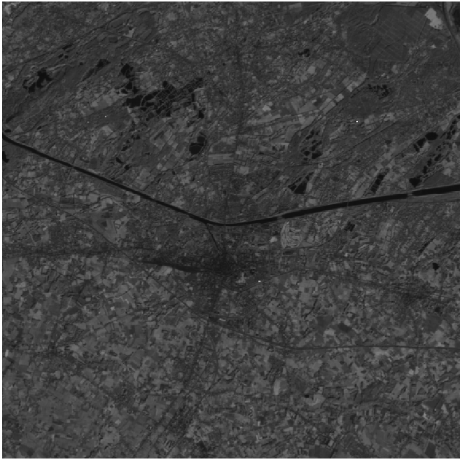
Band Number	2
Spectral Window (nm)	525-605
Spatial Resolution (m)	30
Entropy	4.3416

Data (2000 DFC) - Hasselt, Belgium - Landsat 7



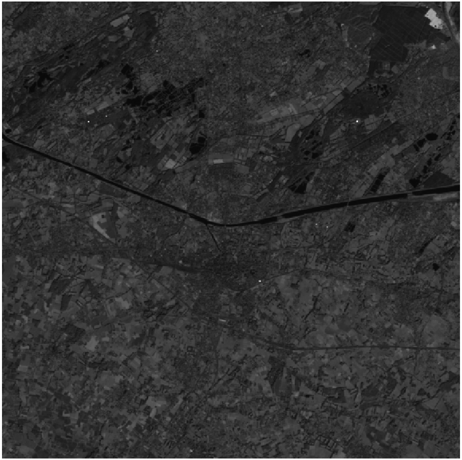
Band Number	3
Spectral Window (nm)	630-690
Spatial Resolution (m)	30
Entropy	4.8394

Data (2000 DFC) - Hasselt, Belgium - Landsat 7



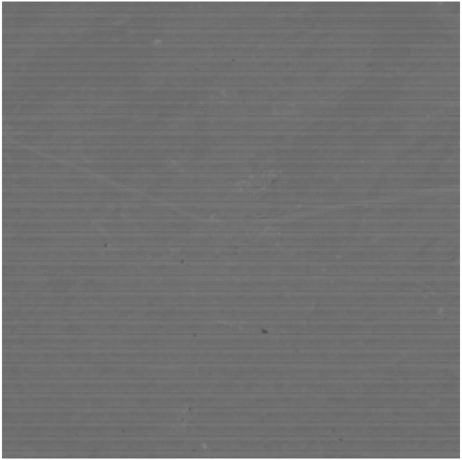
Band Number	4
Spectral Window (nm)	750-900
Spatial Resolution (m)	30
Entropy	6.0074

Data (2000 DFC) - Hasselt, Belgium - Landsat 7



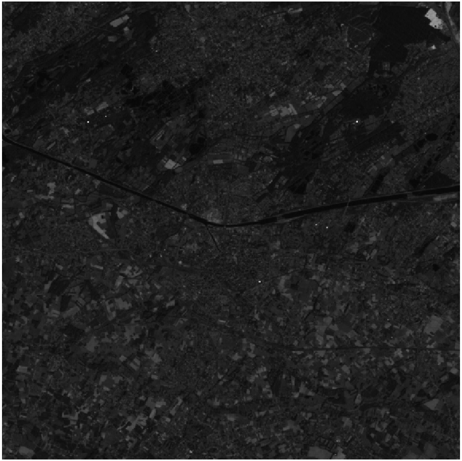
Band Number	5
Spectral Window (nm)	1550-1750
Spatial Resolution (m)	30
Entropy	5.8962

Data (2000 DFC) - Hasselt, Belgium - Landsat 7



Band Number	6
Spectral Window (nm)	1040-1250
Spatial Resolution (m)	60
Entropy	3.5980

Data (2000 DFC) - Hasselt, Belgium - Landsat 7



Band Number	7
Spectral Window (nm)	2090-2350
Spatial Resolution (m)	30
Entropy	5.5004

Data (2000 DFC) - Hasselt, Belgium - Landsat 7

Band Number	8
Spectral Window (nm)	520-900
Spatial Resolution (m)	15
Entropy	4.8442

Fused Image

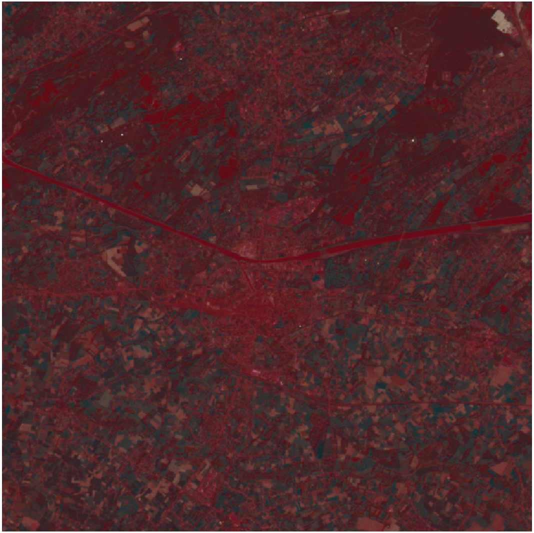


Figure: Multispectral bands fused with panchromatic band, via Wavelet Packet Transform and Principal Component Analysis

Thank you for your time!